Abstract

Counting erasure patters of SPC product codes by means of bipartite graphs

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The single parity-check (SPC) code is one of the most popular MDS error detection codes, since it is very easy to implement [1]. One bit is appended to an information sequence of length $n-1$, such that the resultant codeword has an even number of ones. Two or more SPC codes can be used jointly to obtain an SPC product code. This product code has 4 as minimum distance, then it can recover all erasure patterns with one, two, and three erasures. However, up to $2n-1$ erasures can be corrected in some special cases. Furthermore, a codeword of length n^2 can be represented by an erasure pattern of size $n \times n$, where the unique information considered is the position of the erasures. In [1], authors proposed an approach of the post-decoding erasure rate of the SPC product code. This process was based on observing the structure of the erasure patterns, classifying them into correctable or uncorrectable. In this work, we represent each erasure pattern by a binary matrix where there is a 1 in the position of the erasures. Then, the problem of counting patterns can be seen as a problem of counting binary matrices with some special properties. At the same time, we can represent each erasure pattern by a bipartite graph [2] with n nodes in each vertex class and the same number of edges as erasures. The binary matrix mentioned before is the bi-adjacency matrix of the bipartite graph. Then, the problem of counting uncorrectable erasure patterns can be seen as a problem of counting bipartite graphs with cycles. In [3], the author used Kostka number to count binary matrix with a fixed row and column sum. Here, we use the same idea to provide an expression that helps to count the number of bipartite graphs with cycles and, therefore, to count the number of strict uncorrectable erasure patterns.

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References

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