Abstract

Generalizations of Heffter arrays and biembedding (multi)graphs on surfaces

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In this talk, we present two classes of partially filled arrays that generalize the concept of Heffter array introduced by Archdeacon in 2015.

First of all, we introduce the relative Heffter arrays here denoted by $H_t(m, n; s, k)$. A $H_t(m, n; s, k)$ is an $m \times n$ partially filled array with elements in \mathbb{Z}_v , where v = 2nk + t, whose rows contain s filled cells and whose columns contain k filled cells, such that the elements in every row and column sum to zero and, for every $x \in \mathbb{Z}_v$ not belonging to the subgroup of order t, either x or -x appears in the array. Then we present the more general class of λ -fold relative Heffter arrays denoted by ${}^{\lambda}H_t(m,n;s,k)$. In this case $v = \frac{2nk}{\lambda} + t$ and, given an element $x \in \mathbb{Z}_v$ that does not belong to the subgroup of order t, the sum of the occurrences of x and -x in the array is required to be λ .

Finally, we show that also these generalizations of the Heffter arrays, as well as the classical concept, can be used to construct 2-colourable embeddings (i.e. biembeddings) of cyclic cycle decompositions of complete multipartite (multi)graphs into orientable surfaces.

Joint work with Fiorenza Morini, Anita Pasotti and Marco Antonio Pellegrini.