Abstract

Constructions of divisible design Cayley graphs

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A k-regular graph on v vertices is a *divisible design graph* with parameters $(v, k, \lambda_1, \lambda_2, m, n)$ if the vertex set can be partitioned into m classes of size n, such that two distinct vertices from the same class have exactly λ_1 common neighbours, and two vertices from different classes have exactly λ_2 common neighbours [1, 3]. This partition into classes is called a *canonical partition.*

Divisible design graphs are a special case of Deza graphs. A *Deza graph* with parameters (v, k, b, a) is a k-regular graph on v vertices in which the number of common neighbours of two distinct vertices takes exactly two values a or b, where $a \leq b$. Deza graphs were introduced in [2]. Deza graphs generalise strongly regular graphs since the number of common neighbours are independent of vertex adjacency.

Let G be a finite group and S be a subset of G which does not contain the identity element and is closed under inversion. The Cayley graph $\mathrm{Cay}(G, S)$ is a graph with the vertex set G in which two vertices x, y are adjacent if and only if $xy^{-1} \in S$.

The following theorem gives necessary and sufficient conditions for a Deza Cayley graph to be a divisible design Cayley graph.

Theorem. *Let* $\mathrm{Cay}(G, S)$ *be a Deza graph with parameters* (v, k, b, a). *Let* A, B *and* $\{e\}$ *be a partition of* G *and* SS^{-1} *be a multiset such that* $SS^{-1} = aA + bB + k\{e\}$. *If either* $A \cup \{e\}$ *or* $B \cup \{e\}$ *is a subgroup of* G, *then* $\mathrm{Cay}(G, S)$ *is a divisible design graph and the right cosets of this subgroup give a canonical partition of the graph.*

Conversely, if $\mathrm{Cay}(G, S)$ *is a divisible design graph, then the class of its canonical partition which contains the identity of* G *is a subgroup of* G *and classes of the canonical partition of the divisible design graph coincide with the cosets of this subgroup.*

This Theorem shows that Cayley divisible design graphs arise only by means of divisible difference sets relative to some subgroup. Using divisible difference sets relative to some subgroup we present new constructions of divisible design graphs. This is joint work with Leonid Salaginov.

References

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