Abstract

Completely regular codes in Johnson and Grassmann graphs with small covering radii

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Let C be a code (a collection of vertices) in a regular graph Γ. A vertex x is in C_i if the minimum of the distances between x and the vertices of C is i. The maximum of these distances is called *the covering radius* of C and is denoted by ρ. A code C is called *completely regular* if there are numbers $\alpha_0, \ldots, \alpha_\rho$, $\beta_0, \ldots, \beta_{\rho-1}$, $\gamma_1, \ldots, \gamma_\rho$ such that for every $i \in \{0, \ldots, \rho\}$ any vertex of C_i is adjacent to exactly α_i, β_i and γ_i vertices of C_{i-1}, C_i and C_{i+1} respectively.

Let $\mathcal{L}$ be a 2-spread in $PG(n-1, q)$ (i.e. $1-(n, 2, 1)_q$–design). This code is known to be completely regular in the Grassmann graph $J_q(n, 2)$ [1,Corollary 3.5] with covering radius 1. For fixed $k, k \geq 3$ consider the code D of k-subspaces, which do not contain subspaces from $\mathcal{L}$. When k is 3 this code is completely regular in $J_q(n, 3)$ [2]. Our result is that if k is 4 and $\mathcal{L}$ is a Desarguesian spread then the code D is completely regular in $J_q(n, 4)$ with covering radius 2. In a similar manner we construct a completely regular code in the Johnson graph $J(n, 6)$ from the affine Steiner quadruple system of order $n = 2^m$. We also obtain several new completely regular codes with covering radius 1 in the Grassmann graph $J_2(6, 3)$ using binary linear programming. A detailed description of these results with proofs could be found in [3].

[1] W.J. Martin, Completely regular designs, J. Combin. Des. 4 (1998) 261–273.

[2] S. De Winter, K. Metsch, Perfect 2-Colorings of the Grassmann Graph of Planes, Electronic Journal of Combinatorics, Volume 27, Issue 1, P1.21 (2020).

[3] Ivan Mogilnykh, Completely regular codes in Johnson and Grassmann graphs with small covering radii, https://arxiv.org/abs/2012.06970.