Abstract

On flag-transitive symmetric 2-(v, k, λ) designs

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In literature there are many papers devoted to the study of symmetric 2-(v, k, λ) designs $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ admitting a flag-transitive automorphism group G. If $\lambda = 1$, then G acts point-primitively on $\mathcal{D}$, and both $\mathcal{D}$ and G are essentially classified by Kantor (1985). The case $\lambda > 1$ is different: it is far from being settled and there are several examples of 2-designs admitting a flag-transitive, point-imprimitive automorphism group. In the latter case, if G acts point-primitively on $\mathcal{D}$, remarkable results were obtained by O'Reilly-Reguerio (2005) for $\lambda = 2$, Zhou et al. (2010) for $\lambda = 3, 4$, by Braić et al. (2011) for $v < 2500$, by Biliotti and Montinaro (2016) and by Alavi, Zhou et al. (2021) for $gcd(k, v) = 1$. If G acts point-imprimitively on $\mathcal{D}$, then Praeger and Zhou (2006) have shown that each line of $\mathcal{D}$ intersects any block of imprimitivity either in 0 or in a constant number x of points, $x \geq 2$. Moreover, either $k \leq \lambda(\lambda - 3)/2$ or the parameters of $\mathcal{D}$ are determined as a function of λ.

The aim of this talk is to provide an approach to determine $\mathcal{D}$ for $k > \lambda(\lambda - 3)/2$ and $x > 2$. More precisely we show that, for $x > 2$ the incidence structure $\mathcal{D}_0 = (\mathcal{P}_0, \mathcal{B}_0)$, where $\mathcal{P}_0$ is a block of imprimitivity for G on $\mathcal{P}$ and $\mathcal{B}_0 = \{\mathcal{P}_0 \cap \ell \neq \varnothing : \ell \in \mathcal{B}\}$, is a 2-design admitting $G_{\mathcal{P}_0}^{\mathcal{P}_0}$ as a flag-transitive automorphism group. Also, the group $G_{\mathcal{P}_0}^{\mathcal{P}_0}$ acts point-primitively on $\mathcal{D}$ for $k > \lambda(\lambda - 3)/2$. This allows to classify $\mathcal{D}_0$ and hence to determine strong constraints for the structure of $\mathcal{D}$.