Abstract

A family of non-Cayley cores that is constructed from vertex-transitive or strongly regular self-complementary graphs

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Let Γ be a finite simple graph on n vertices. In the talk I will consider the graph $\Gamma \equiv \bar{\Gamma}$ on $2n$ vertices, which is obtained as the disjoint union of Γ and its complement $\bar{\Gamma}$, where we add a perfect matching such that each its edge joins two copies of the same vertex in Γ and $\bar{\Gamma}$. The graph $\Gamma \equiv \bar{\Gamma}$ generalizes the Petersen graph, which is obtained if Γ is the pentagon. It is a non-Cayley graph if $n > 1$, and is vertex-transitive if and only if Γ is vertex-transitive and self-complementary. In this case $\Gamma \equiv \bar{\Gamma}$ is Hamiltonian-connected whenever $n > 5$. It is shown that the fraction between the cardinalities of the automorphism groups of $\Gamma \equiv \bar{\Gamma}$ and Γ can attain only values 1, 2, 4, or 12, and the corresponding four classes of graphs are described. The spectrum of the adjacency matrix of $\Gamma \equiv \bar{\Gamma}$ is computed whenever Γ is regular. The main results involve the endomorphisms of $\Gamma \equiv \bar{\Gamma}$. It is shown that the graph $\Gamma \equiv \bar{\Gamma}$ is a core, i.e. all its endomorphisms are automorphisms, whenever Γ is strongly regular and self-complementary. The same result is obtained for many cases, where Γ is vertex-transitive and self-complementary.