Abstract

Small complete caps in $PG(4n+1, q)$

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Let $PG(r, q)$ denote the r-dimensional projective space over the finite field with q elements F_q. A *k-cap* of $PG(r, q)$ is a set of k points no three of which are collinear. A k-cap of $PG(r, q)$ is said to be *complete* if it is not contained in a $(k+1)$-cap of $PG(r, q)$. The study of caps is not only of geometrical interest, but arises from coding theory. Indeed, by identifying the representatives of the points of a complete k-cap of $PG(r, q)$ with columns of a parity check matrix of a q–ary linear code, it follows that (apart from three sporadic exceptions) complete k-caps of $PG(r, q)$ with $k > r+1$ and non-extendable linear $[k, k-r-1, 4]_q$ 2–codes are equivalent objects.

One of the main issue is to determine the spectrum of the sizes of complete caps in a given projective space and in particular their maximal and minimal possible values. For the size $t_2(r, q)$ of the smallest complete cap in $PG(r, q)$, the trivial lower bound is $t_2(r, q) > \sqrt{2}q^{\frac{r-1}{2}}$. Apart from the cases q even and r odd, all known infinite families of complete caps explicitly constructed in $PG(r, q)$ have size far from the trivial bound.

In this talk I will describe the construction of a complete cap of $PG(4n+1, q)$ of size $2(q^{2n} + \cdots + 1)$ that is obtained by projecting two disjoint Veronese varieties of $PG(n(2n+3), q)$ from a suitable $(2n^2-n-2)$-dimensional projective space. This establishes that the trivial lower bound on $t_2(4n+1, q)$ is essentially sharp.

This is joint work with A. Cossidente, B. Csajbók and G. Marino.