Abstract

Reed – Muller like codes and their intersections

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The classical binary *Reed – Muller code of order* r, $0 \leq r \leq m$, for any $m \geq 1$ is defined as the set of all vectors of length 2^m corresponding to the boolean functions of m variables of degree not more than r. The Reed – Muller code is linear and has the following parameters: the length n of the code is 2^m, the size $2^k, k = \sum_{i=0}^{r} \binom{m}{i}$ and the *code distance* (the minimum value of the Hamming distance between any two different codewords from the code) is 2^{m-r}.

The code is called *self-complementary* if for any codeword x the code contains the vector $x + \mathbf{1}^n$, where $\mathbf{1}^n$ is the all-one vector of length n. The Reed – Muller code is self-complementary. A binary self-complementary code with the parameters of the classical Reed – Muller code is called a *Reed – Muller like code*. Such code is not necessarily linear. The class of the Reed – Muller like codes contains a rich families of codes obtained in [1]–[3].

We prove that there exist two Reed – Muller like codes of order r of length 2^m with the intersection number equaled $2\eta_1\eta_2$, where $1 \leq \eta_s \leq |RM(r-1, m-1)|$, $s \in \{1, 2\}$ for any admissible length beginning with 16. The result generalizes the result [4] concerning the intersection problem for perfect binary codes.

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