A family of non-Cayley cores that is constructed from vertex-transitive or strongly regular self-complementary graphs

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Γ finite simple/undirected graph with $|V(\Gamma)| \geq 1$

Λ finite simple/undirected graph with $|V(\Lambda)| \geq 0$

A *homomorphism* between graphs Γ_1 and Γ_2 is a map $\Phi : V(\Gamma_1) \to V(\Gamma_2)$ such that

$$\{u, v\} \in E(\Gamma_1) \Longrightarrow \{\Phi(u), \Phi(v)\} \in E(\Gamma_2).$$

If $\Gamma_1 = \Gamma_2$, then homomorphism = *endomorphism*.

Cores

A graph is a *core* if all its endomorphisms are automorphisms.

Basic examples:

- complete graphs
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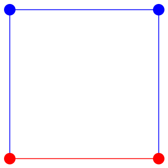
Example: $\mathrm{core}(C_4) = K_2$



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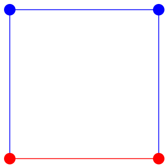

Proposition (cf. Godsil & Royle)

Every graph Γ has a core, which is an induced subgraph and is unique up to isomorphism.

Many 'nice' graphs are cores or their cores are complete

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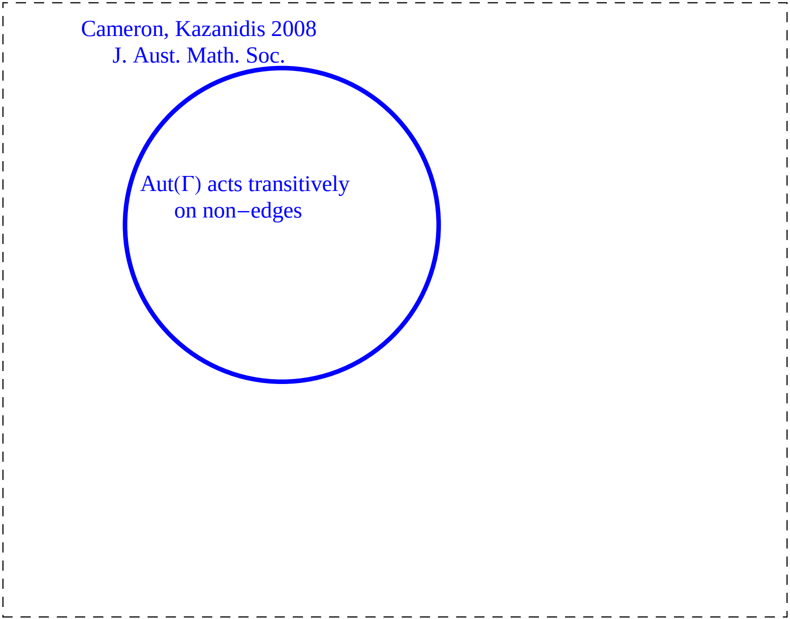

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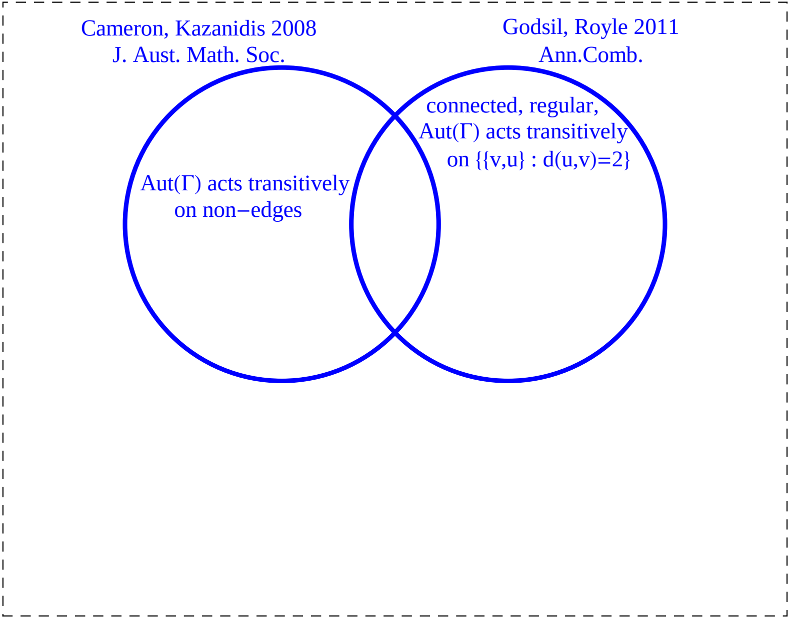

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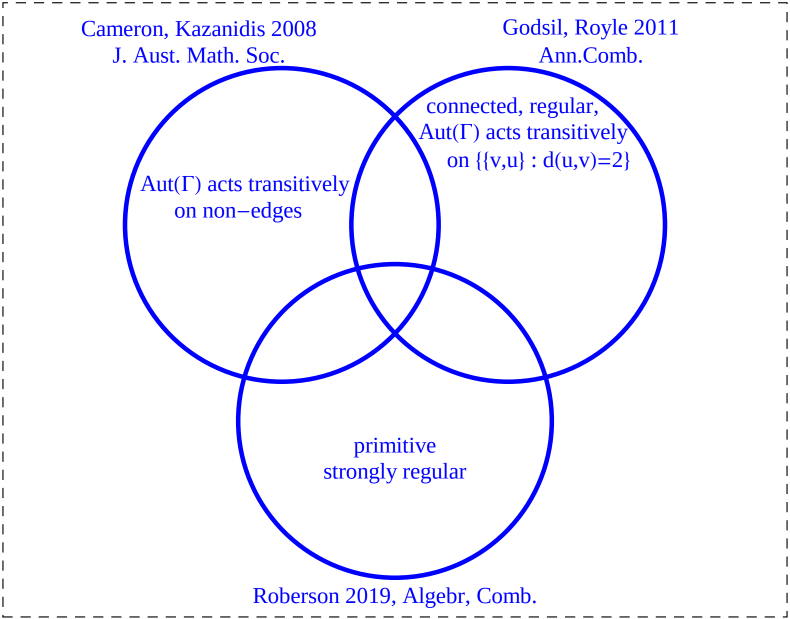

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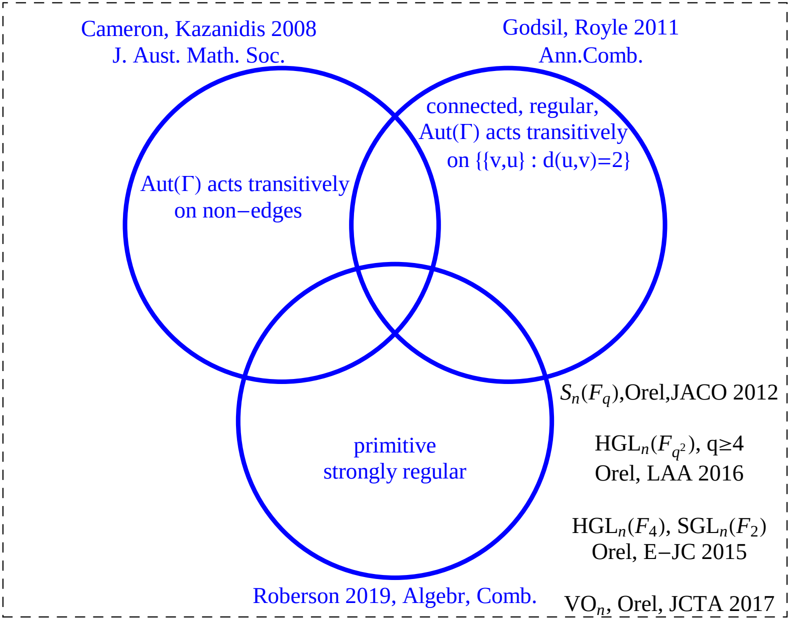
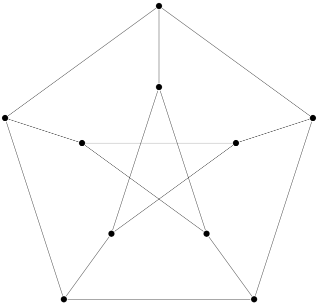
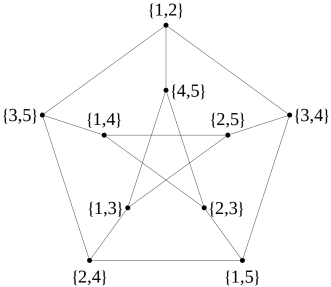


A famous core: the Petersen graph



A generalization: Kneser graphs $K(v, r)$

$$V(K(v,r)) = \{S \subseteq \{1,\ldots,v\} : |S| = r\}$$
$$E(K(v,r)) = \{\{S_1, S_2\} : S_1 \cap S_2 = \emptyset\}$$



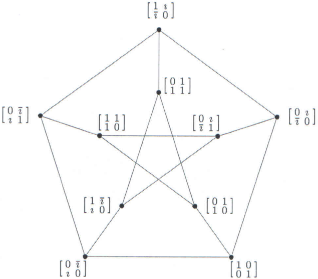
cf. Godsil, Royle 2001

If $v > 2r$, then $K(v, r)$ is a core.

Another generalization: $HGL_n(\mathbb{F}_4)$

$$\mathbb{F}_4 = \{0, 1, \imath, 1+\imath\} = \mathbb{F}_2 + \imath\mathbb{F}_2, \quad \imath^2 = 1 + \imath = \bar{\imath}$$

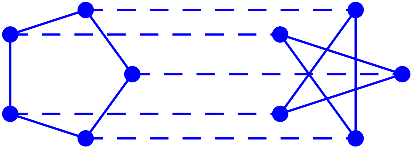
$$V(HGL_n(\mathbb{F}_4)) = \{\text{invertible Hermitian } n \times n \text{ matrices over } \mathbb{F}_4\}$$
$$E(HGL_n(\mathbb{F}_4)) = \{\{A_1, A_2\} : \operatorname{rank}(A_1 - A_2) = 1\}$$



Orel, E-JC 2015

If $n \geq 2$, then $HGL_n(\mathbb{F}_4)$ is a core.

Yet another generalization: complementary prism $\Gamma\bar{\Gamma}$



(my former notation: $\Gamma \equiv \bar{\Gamma}$)

If Γ is a graph with $V(\Gamma) = \{v_1, \ldots, v_n\}$ let $V(\Gamma\bar{\Gamma}) = W_1 \cup W_2$,

$$W_1 = \{(v_1, 1), \ldots, (v_n, 1)\} \quad \text{and} \quad W_2 = \{(v_1, 2), \ldots, (v_n, 2)\},$$

and let $E(\Gamma\bar{\Gamma})$ be:

$$\begin{gathered}\Big\{\{(u,1),(v,1)\} : \{u,v\} \in E(\Gamma)\Big\} \\ \cup\Big\{\{(u,2),(v,2)\} : \{u,v\} \in E(\bar{\Gamma})\Big\} \\ \cup\Big\{\{(u,1),(u,2)\} : u \in V(\Gamma)\Big\}.\end{gathered}$$

Yet another generalization: $\Gamma\bar{\Gamma}$

Main question

When is $\Gamma\bar{\Gamma}$ a core?

Other problems

- $\mathrm{Aut}(\Gamma\bar{\Gamma})$; related to
  - self-complementary vertex-transitive graphs
  - non-Cayley vertex-transitive graphs
- Hamiltonicity of $\Gamma\bar{\Gamma}$

Already known results: diameter and spectrum

Lemma (Haynes, Henning, Slater, van der Merwe, 2007)

$\Gamma\bar{\Gamma}$ is a connected graph with $\mathrm{diam}(\Gamma\bar{\Gamma}) \leq 3$. Moreover,

$$\mathrm{diam}(\Gamma\bar{\Gamma}) = 1 \iff \Gamma \cong K_1,$$
$$\mathrm{diam}(\Gamma\bar{\Gamma}) = 2 \iff \mathrm{diam}(\Gamma) = 2 = \mathrm{diam}(\bar{\Gamma}).$$

Lemma (Cardoso, Carvalho, de Freitas, Vinagre, 2018)

If Γ is a connected k-regular graph on n vertices with eigenvalues $k = \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$, then the eigenvalues of $\Gamma\bar{\Gamma}$ equal

$$\left\{\frac{n-1 \pm \sqrt{(n-1)^2 - 4((n-k-1)k-1)}}{2}\right\} \cup \left\{\frac{-1 \pm \sqrt{1+4(\lambda_i^2+\lambda_i+1)}}{2} : 2 \leq i \leq n\right\}.$$

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Two special families of graphs: $C_5(\Lambda)$, $A(\Lambda)$

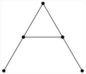
$\mathrm{Aut}(\Gamma) = \{\text{automorphisms of } \Gamma\}$
$\overline{\mathrm{Aut}(\Gamma)} = \{\text{antimorphisms of } \Gamma\} = \{\text{isomorphisms } \Gamma \to \bar{\Gamma}\}$

Definition

Γ is self-complementary (s.c.) if $\overline{\mathrm{Aut}(\Gamma)} \neq \emptyset$

Examples of s.c. graphs

C_5, A-graph



$C_5(\Lambda)$ = constructed from C_5 by replacing one vertex with graph Λ
$A(\Lambda)$ = constructed from A by replacing the 'top' vertex with Λ
FACT: if Λ is s.c., then $C_5(\Lambda)$ and $A(\Lambda)$ are s.c.

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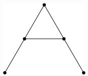

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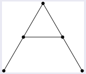

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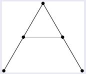

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$\mathrm{Aut}(\Gamma\bar{\Gamma})$

Proposition (Orel 2021+)

If $\Gamma = C_5(\Lambda)$, then $\mathrm{Aut}(\Gamma\bar{\Gamma})$ is isomorphic to

- S_5 if $|V(\Lambda)| = 1$
- $\left(\mathrm{Aut}(\Gamma) \cup \overline{\mathrm{Aut}(\Gamma)}\right) \rtimes \mathbb{Z}_2$ if Λ is s.c. with $|V(\Lambda)| \neq 1$
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Proposition (Orel 2021+)

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$\mathrm{Aut}(\Gamma\bar{\Gamma})$ & corollaries

Corollary (Orel 2021+)

For each graph Γ, $\frac{|\mathrm{Aut}(\Gamma\bar{\Gamma})|}{|\mathrm{Aut}(\Gamma)|} \in \{1, 2, 4, 12\}$.

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$\Gamma\bar{\Gamma}$ is vertex-transitive if and only if Γ is vertex-transitive and s.c.

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It follows from a result of Muzychuk (Bull. London Math. Soc. 1999) that the orders of graphs $\Gamma\bar{\Gamma}$, which are vertex-transitive (and non-Cayley) are precisely the values

$$2p_1^{\alpha_1}\cdots p_s^{\alpha_s}, \text{ where } p_i^{\alpha_i} \equiv 1 \pmod 4) \text{ for all } i$$

($p_1, \ldots, p_s$ are distinct primes, $\alpha_i \geq 1$) or equivalently

$$2\Big((2i)^2 + (2j+1)^2\Big) \quad \text{with} \quad i, j \in \{0, 1, 2, \ldots\} \text{ and } (i, j) \neq (0, 0).$$

$\Gamma\bar{\Gamma}$ is NOT a lexicographic product of two graphs with at least 2 vertices.

Proposition (Orel 2021+)

If Γ is s.c. vertex-transitive graph on $n > 5$ vertices, then $\Gamma\bar{\Gamma}$ is Hamiltonian-connected.

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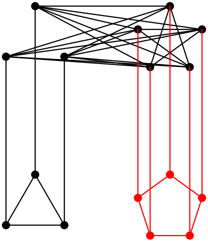
Lemma

If $\Gamma \ncong K_2, \overline{K_2}$, then there are only 5 possibilities for $\mathrm{core}(\Gamma\bar{\Gamma})$:

1. $\Gamma\bar{\Gamma}$ is a core
2. $V\big(\mathrm{core}(\Gamma\bar{\Gamma})\big) \subseteq W_1$ and $\mathrm{core}(\Gamma\bar{\Gamma}) \cong \mathrm{core}(\Gamma)$
3. $V\big(\mathrm{core}(\Gamma\bar{\Gamma})\big) \subseteq W_2$ and $\mathrm{core}(\Gamma\bar{\Gamma}) \cong \mathrm{core}(\bar{\Gamma})$
4. complicated but highly constrained structure
5. complicated but highly constrained structure

Example of possibility (4), (5): $\Gamma = C_3 + C_5$



Corollary (Orel 2021+)

If Γ is $(\frac{n-1}{2})$-regular, where $|V(\Gamma)| = n$, then only possibilities (1), (2), (3) can occur.

The same result is true for each graph Γ with at least 4 vertices if we assume that $\mathrm{core}(\Gamma\bar{\Gamma})$ is regular.

Theorem (Orel 2021+)

If Γ is strongly regular s.c. graph, then $\Gamma\bar{\Gamma}$ is a core.

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If Γ is s.c. vertex-transitive graph, and Γ is either a core or its core is complete, then $\Gamma\bar{\Gamma}$ is a core.

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