Restrictions on parameters of partial difference sets in nonabelian groups

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Why partial *difference* set? Originally interest was in abelian groups, and the operation was addition.

Small example

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- G: additive group of $\mathrm{GF}(13)$
- $S = \{1, 3, 4, 9, 10, 12\}$

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For elements in S:

- $1 = 4 - 3 = 10 - 9$
- $3 = 4 - 1 = 12 - 9$
- $4 = 3 - 12 = 1 - 10$
- $9 = 12 - 3 = 10 - 1$
- $10 = 1 - 4 = 9 - 12$
- $12 = 3 - 4 = 9 - 10$

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For nonidentity elements not in S:

- $2 = 3 - 1 = 12 - 10 = 1 - 12$
- $5 = 9 - 4 = 1 - 9 = 4 - 12$
- $6 = 9 - 3 = 10 - 4 = 3 - 10$
- $7 = 3 - 9 = 4 - 10 = 10 - 3$
- $8 = 4 - 9 = 9 - 1 = 12 - 4$
- $11 = 1 - 3 = 10 - 12 = 12 - 1$

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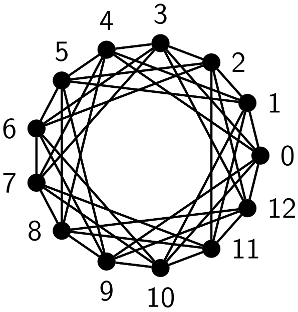
S is a $(13, 6, 2, 3)$-PDS.

Small example, continued

Example

- G: additive group of of $\mathrm{GF}(13)$
- $S = \{1, 3, 4, 9, 10, 12\}$
- S is a $(13, 6, 2, 3)$-PDS with $0 \notin S$, $S = -S$
- $\mathrm{Cay}(G, S)$: undirected $(13, 6, 2, 3)$-strongly regular Cayley graph.



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A (v, k, λ, μ)-PDS is called *regular* if $1 \notin S$ and $S = S^{-1}$.

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G: finite group

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The group G acts *regularly* on the set Ω if G acts transitively and fixed-point freely (other than the identity).

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SO: regular (v, k, λ, μ)-PDS in $G \leftrightarrow G$ acts regularly on vertices of (v, k, λ, μ)-SRG

What's known?

- Extensive knowledge for abelian groups (see Ma [Ma94])
- Very few known for nonabelian groups!

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- Extensive knowledge for abelian groups (see Ma [Ma94])
- Very few known for nonabelian groups!
- **Smith [Smi95]:** regular $(4t^2, 2t^2 - t, t^2 - t, t^2 - t)$-PDS's in certain nonabelian groups
- **Kantor [Kan86], Ghinelli [Ghi12]:** regular $(q^3, q^2 + q - 2, q - 2, q + 2)$-PDS in Heisenberg group of order q^3 (q odd prime power)
- **S. [Swa15]:** regular $(p^3, p^2 + p - 2, p - 2, p + 2)$-PDS S of extraspecial group of order p^3, exponent p^2 (p odd)
- **Feng, He, Chen [FHC20]:** PDS's of exponent 4, 8, and 16 and of nilpotency class 2, 3, 4, and 6

Generalized Quadrangles

A generalized quadrangle (GQ) is a point-line incidence geometry.

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- **Feng, Li [FL21]:** Finite groups acting regularly on points of a finite generalized quadrangle can have unbounded nilpotency class(!!!)

Motivation: Yoshiara's Restrictions

Theorem (Yoshiara [Yos07])

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- $d_1(x) = |x^G \cap \Delta||C_G(x)| \equiv (s+1)(t+1) \pmod{s+t}$;
- $\gcd(s, t) > 1 \Rightarrow x^G \cap \Delta \neq \varnothing$.

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Then,

$$-\nu_3 d_0(x) + d_1(x) \equiv \mu - \nu_3(\nu_2+1) \pmod{(\nu_2-\nu_3)}.$$

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- $d_1(x) = |x^G \cap \Delta||C_G(x)| \equiv \mu - \nu_3(\nu_2 + 1) \pmod{\nu_2 - \nu_3}$;
- *If $(\nu_2 - \nu_3)$ does not divide $\mu - \nu_3(\nu_2 + 1)$, then $x^G \cap \Delta \neq \varnothing$.*

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If $\nu_2 - \nu_3$ divides neither $\mu - \nu_3(\nu_2 + 1)$ nor $v - 2k + \lambda - \nu_3(\nu_2 + 1)$, then $x^G \cap \Delta \neq \varnothing$ and $x^G \cap \Delta^c \neq \varnothing$.

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Proof.

Apply the previous lemma to both Γ *and* its complement Γ^c! $\square$

New Results, Cont.

Let

$$\nu_2 = \frac{1}{2}\left(\lambda - \mu + \sqrt{\Lambda}\right), \quad \nu_3 = \frac{1}{2}\left(\lambda - \mu - \sqrt{\Lambda}\right) \in \mathbb{Z},$$

where $\Lambda = (\lambda - \mu)^2 + 4(k - \mu) = (\nu_2 - \nu_3)^2$.

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Corollary (S. Tauscheck [ST21])

Suppose

- $|G| = v$;

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Suppose

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- *G has a nontrivial center (EX: G is a p-group);*
- $\nu_2 - \nu_3$ *divides neither of* $\mu - \nu_3(\nu_2 + 1)$ *or* $v - 2k + \lambda - \nu_3(\nu_2 + 1)$.

Then, G does not contain a regular (v, k, λ, μ)*-PDS.*

Parameters Ruled Out

v	k	λ	μ	ν_2	ν_3	$\nu_2 - \nu_3$	$\mu - \nu_3(\nu_2 + 1)$
28	12	6	4	4	-2	6	14
	15	6	10	1	-5	6	20
63	30	13	15	3	-5	8	35
	32	16	16	4	-4	8	36
88	27	6	9	3	-6	9	33
	60	41	40	5	-4	9	64
105	26	13	4	11	-2	13	28
	78	55	66	1	-12	13	90
105	32	4	12	2	-10	12	42
	72	51	45	9	-3	12	75
105	52	21	30	2	-11	13	63
	52	29	22	10	-3	13	55
117	36	15	9	9	-3	12	39
	80	52	60	2	-10	12	90
176	25	0	4	3	-7	10	32
	150	128	126	6	-4	10	154

Table: Parameters with $v \leqslant 300$ ruled out

Parameters Ruled Out, Cont.

v	k	λ	μ	ν_2	ν_3	$\nu_2 - \nu_3$	$\mu - \nu_3(\nu_2 + 1)$
176	45	18	9	12	-3	15	48
	130	93	104	2	-13	15	143
176	70	18	34	2	-18	20	88
	105	68	54	17	-3	20	108
176	70	24	30	4	-10	14	80
	105	64	60	9	-5	14	110
189	48	12	12	6	-6	12	54
	140	103	105	5	-7	12	147
195	96	46	48	6	-8	14	104
	98	49	49	7	-7	14	105
208	75	30	25	10	-5	15	80
	132	81	88	4	-11	15	143
208	81	24	36	3	-15	18	88
	126	80	70	14	-4	18	130
225	96	51	33	21	-3	24	99
	128	64	84	2	-22	24	150

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v	k	λ	μ	ν_2	ν_3	$\nu_2 - \nu_3$	$\mu - \nu_3(\nu_2 + 1)$
231	30	9	3	9	-3	12	33
	200	172	180	2	-10	12	210
231	40	20	4	18	-2	20	42
	190	153	171	1	-19	20	209
231	90	33	36	6	-9	15	99
	140	85	84	8	-7	15	147
232	33	2	5	4	-7	11	40
	198	169	168	6	-5	11	203
232	63	14	18	5	-9	14	72
	168	122	120	8	-6	14	174
232	77	36	20	19	-3	22	80
	154	96	114	2	-20	22	174
232	81	30	27	9	-6	15	87
	150	95	100	5	-10	15	160
236	55	18	11	11	-4	15	59
	180	135	144	3	-12	15	192

Table: Parameters with $v \leqslant 300$ ruled out

Parameters Ruled Out, Cont.

v	k	λ	μ	ν_2	ν_3	$\nu_2 - \nu_3$	$\mu - \nu_3(\nu_2 + 1)$
275	112	30	56	2	-28	30	140
	162	105	81	27	-3	30	165
279	128	52	64	4	-16	20	144
	150	85	75	15	-5	20	155
285	64	8	16	4	-12	16	76
	220	171	165	11	-5	16	225
297	128	64	48	20	-4	24	132
	168	87	105	3	-21	24	189

Table: Parameters with $v \leqslant 300$ ruled out

Parameters Ruled Out, Cont.

v	k	λ	μ	ν_2	ν_3	$\nu_2 - \nu_3$	$\mu - \nu_3(\nu_2 + 1)$
343	102	21	34	4	-17	21	119
	240	171	160	16	-5	21	245
343	114	45	34	16	-5	21	119
	228	147	160	4	-17	21	245
625	246	119	82	41	-4	45	250
	378	213	252	3	-42	45	420
729	208	37	68	4	-35	39	243
	520	379	350	34	-5	39	525
729	248	67	93	5	-31	36	279
	480	324	300	30	-6	36	486
729	280	127	95	37	-5	42	285
	448	262	296	4	-38	42	486

Table: Prime-power parameters with $v \leqslant 1000$ ruled out

Thanks!

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