On Automorphisms of a binary Fano plane

Kristijan Tabak

Rochester Institute of Technology, Croatia Campus

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It is still unknown if a 2-analog of a Fano plane exists.

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if every $T \in E_{2^2}[E_{2^7}]$ is contained in exactly one $H \in \mathcal{H}$.

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$|\{x_i x_i^\beta \mid i \in [2^{n-k}]\}| = 2^{n-k}$ and $\{x_i x_i^\beta \mid i \in [2^{n-k}]\} \subseteq F$.

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Theorem 0.3 *Let* $\beta \in Aut(E_{2^6})$ *be of order* 2 *with* 31 *fixed point. Then, there is no* E_{2^2} *tiling of* E_{2^6} *such that* $\sum_{i=1}^{2a+1} A_i + \sum_{j=1}^{b} B_j^{\langle \beta \rangle} = E_{2^6} + 20$ *where* $A_i^\beta = A_i,\ B_j \cap B_j^\beta = 1$, *and* $A_i \cong B_j \cong E_{2^2}$.

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Proof: Assume the opposite. Let $\alpha \in Aut(\mathcal{H})$ such that $o(\alpha) = 2$ and $|Fix(\alpha)| = 63$. Let $F = 1 + Fix(\alpha) \cong E_{2^6}$. Take some $H \in \mathcal{H}$ such that $H^\alpha \neq H$. Then $H \not\leq F$ and $T = H \cap F = E_{2^2}$. Therefore, $T^\alpha = H^\alpha \cap F = T$. Thus, $T \leq H \cap H^\alpha$. Hence, $T \cong E_{2^2}$ is a subgroup of two different blocks from $\mathcal{H}$. By the definition of $\mathcal{H}$, that is not possible. Thus, $\alpha/\mathcal{H} = id$. We will argue that this is not possible as well. Take $c \neq 1$ and $\mathcal{H}_c = \sum_{c \in H \in \mathcal{H}} H$. Since $H^\alpha = H$ for all $H \in \mathcal{H}_c$, then $c^\alpha = c$. Then $\alpha = id$ which is a contradiction with $o(\alpha) = 2$. $\square$

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Corollary 0.5 *If $\alpha \in Aut(\mathcal{H})$ is of order 2, then $|Fix(\alpha)| = 15$.*

Theorem 0.6 *Let* $\alpha \in Aut(\mathcal{H})$ *is of order* 4. *Then there are* 28 α-*orbits on* E_{2^7} *of a size* 4. *Furthermore,* $Fix(\alpha^2) = Fix(\alpha) + \sum_{i=1}^{a_2} x_i^{\langle\alpha\rangle}$, *where* a_2 *is the number of* α-*orbits on* E_{2^7} *of a size* 2.

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Remark: Since $2^k + 2a_2 = 2^4$, we get $k \leq 4$. From a class equation and assumption $o(\alpha) = 4$, we can see that α must have a fixed point from $E_{2^7}^*$. Therefore, $|F_1| = 2^k > 1$. This means that we need to analyze cases $k = 1, 2, 3, 4$.

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From $Fix(\alpha) = Fix(\alpha^2)$ we get a contradiction.

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A long and the most difficult case.

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